Masses distribution in a boat

Effects and method of control

Pendulum Test

In the IYRU Permanent Committee's opinion, the distribution of mass should be controlled in a One Design class :
" IYRU 1967 Amendments to the rules of the International Finn Class".

INTR0DUCTI0N

This memorandum comprises two parts

In the first one, I have tried to show that the distribution of masses or matter in a boat plays a dominant part in the losses of its propulsive power. Will the reader, please, forgive me for resorting to mathematical demonstrations; if I am not making myself clear, he may go straight over to my conclusions !

In the second part, I explain the measuring method we have adopted after many trials.

Gilbert LAMBOLEY
may 1971
Revised may 2003

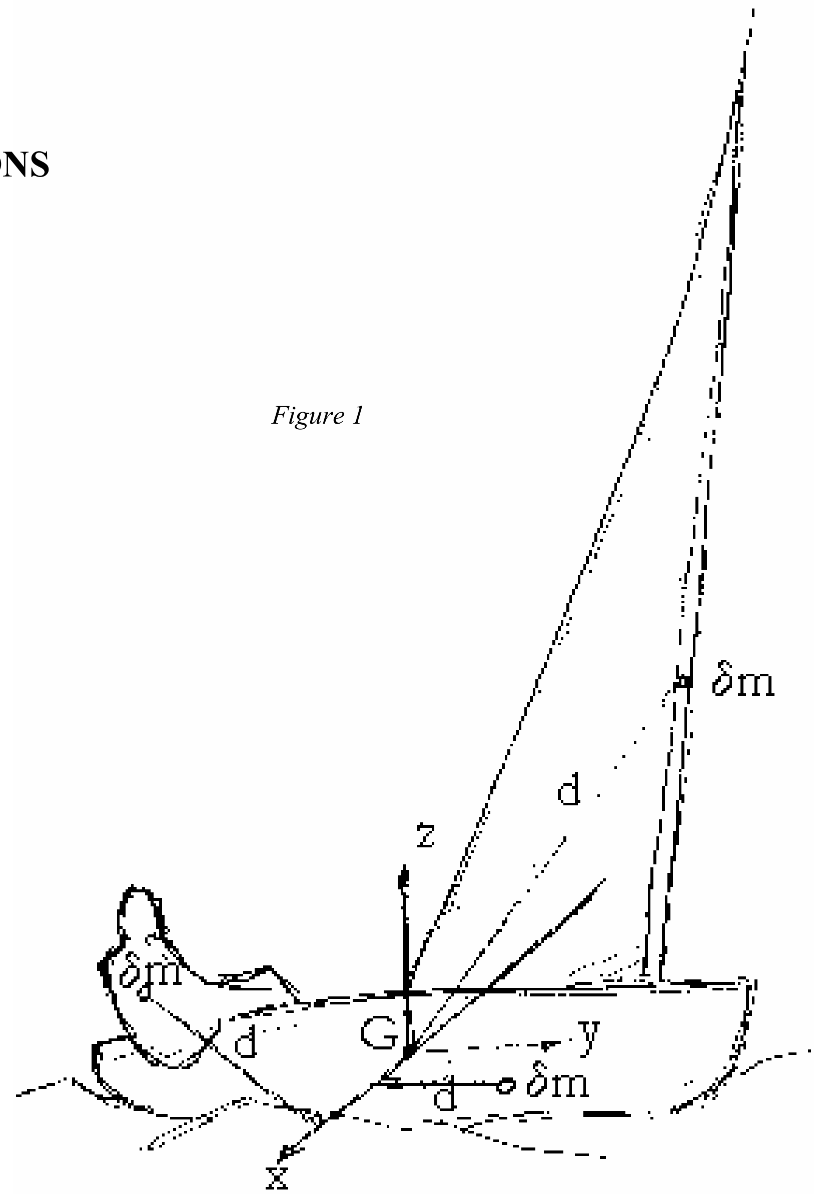
PS The expression "weight distribution" is scientifically incorrect; "mass distribution" or "matter distribution" must be used ; weight is the effort applied to mass by gravity only ; in this paper we shall examine the effects of other accelerations than vector gravity ***g***.

I PART ONE : THEORETICAL ANALYSIS

EFFECTS of the DISTRIBUTION of MASSES

I – 1, DEFINITIONS

Figure 1



In *Fig 1,* if **G** is the centre of gravity, the boat may be considered as a large number of small units of mass **δm** ; if **d** is their distance from the transverse axis **Gx** passing through the centre of gravity **G**, one usually call:

m, total mass of the boat $m = \sum \delta m$

Moment of masses inertia around that axis, the sum of all $\delta m.d^2$ terms, i.e. $I_{Gx} = \sum \delta m.d^2$

Radius of gyration around the same axis, a length r_x such that $I_{Gx} = \sum m r_x^2$

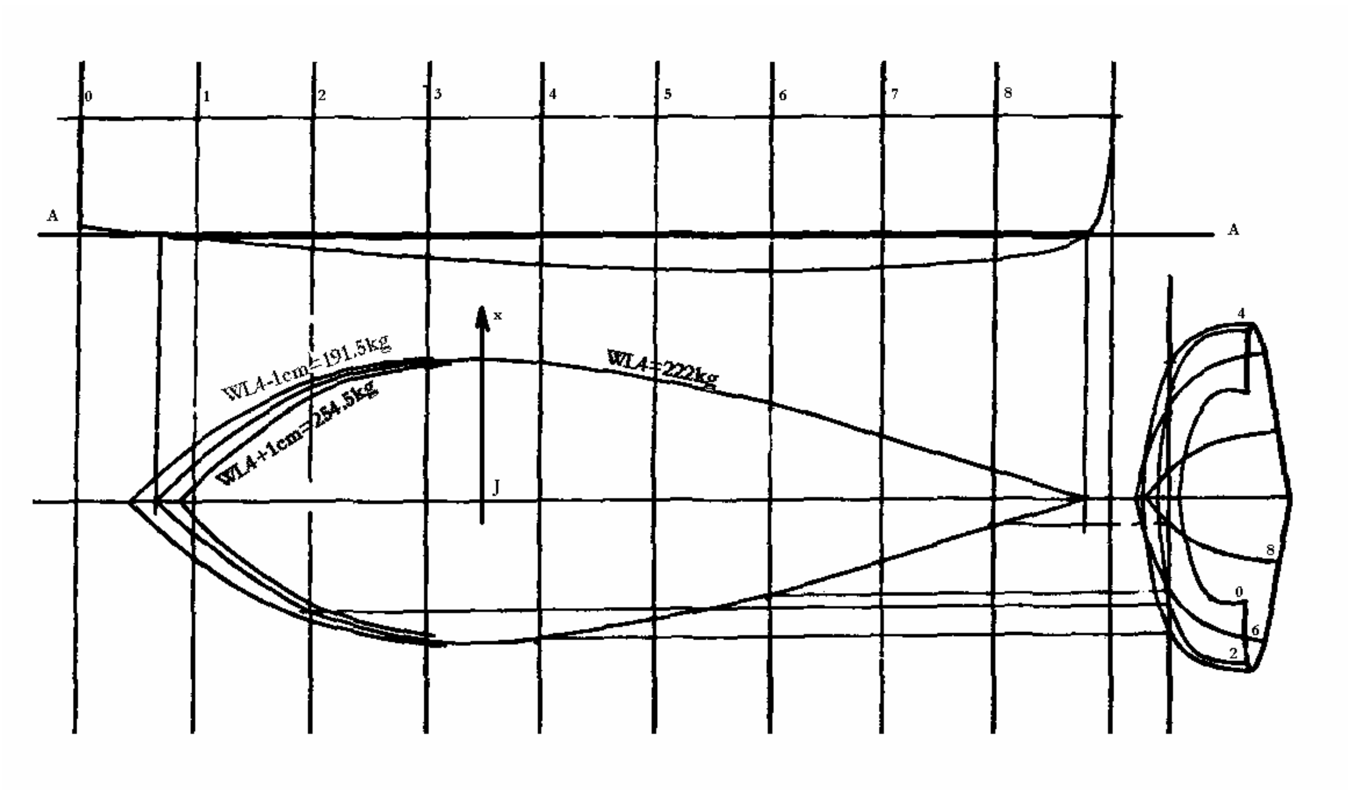
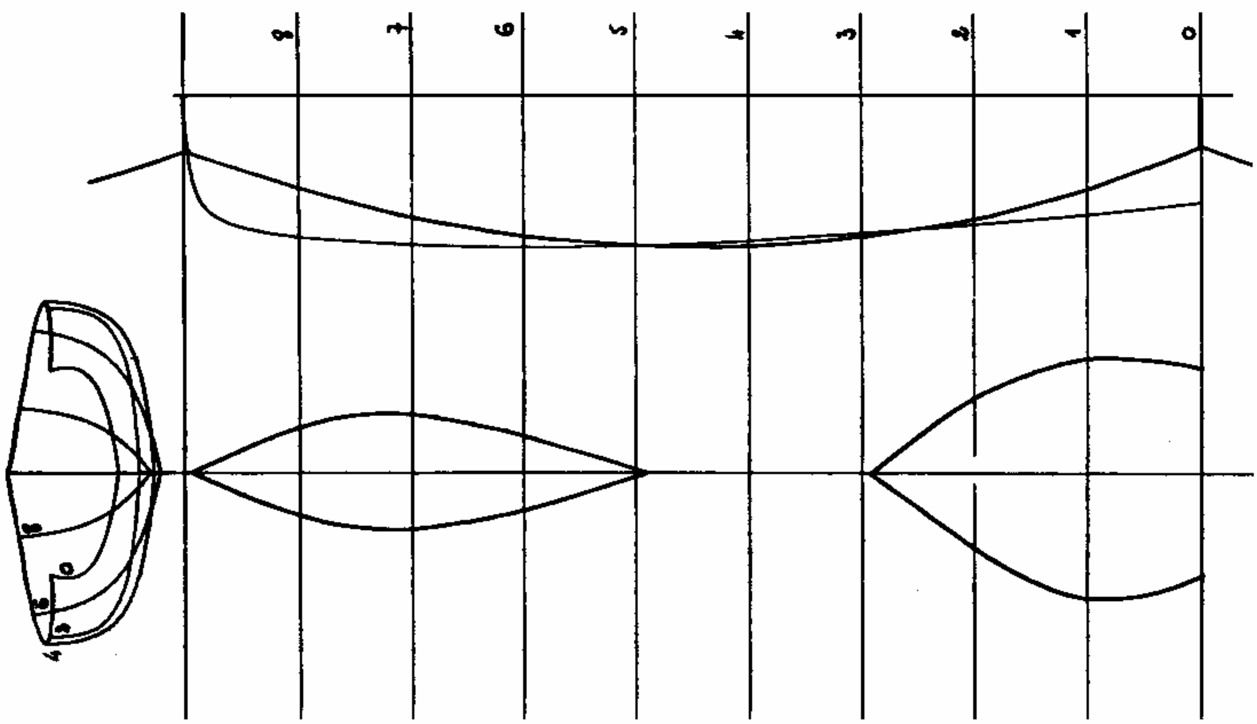


Figure 2



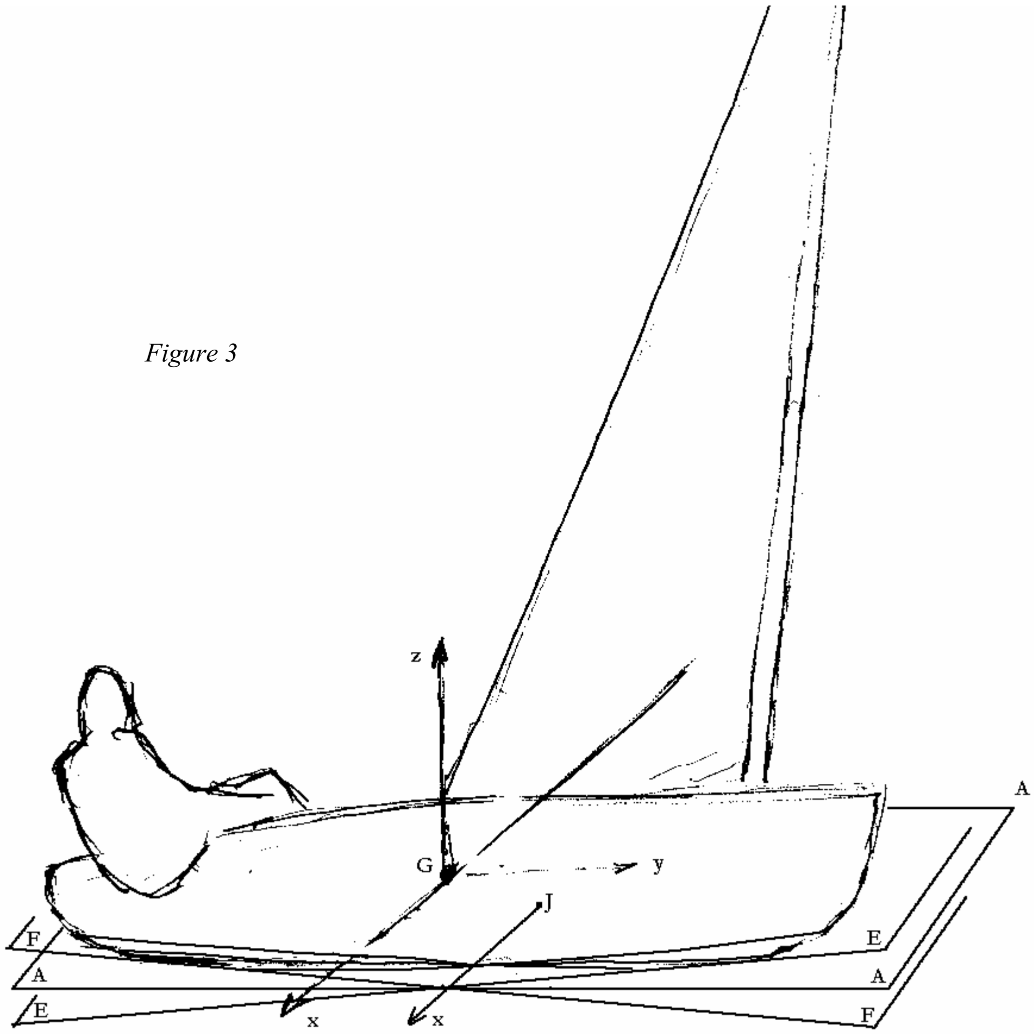
Floatation area, the area inside waterline : **S**
Geometrical inertia of floatation area : J_x or J_y
Distance between centre of buoyancy and metacentre : ρ such that $\rho = \sqrt{J/S}$

Figure 2 above, second part of which is quite an approximation, shows how those may be altered by sea state.

I – 2, BOAT'S MOVEMENT

This movement may be split into six elementary ones : three translations and three rotations,

Figure 3



Sideways drift or **sway** along axis **Gx**,
Forward motion or **surge** along axis **Gy**,
Up and down motion or **heave** along axis **Gz**, between crest and bottom of waves for instance,
Pitching around axis **Gx**,
Heeling or **rolling** around axis **Gy**,
Heading changes or **yaw** around axis **Gz**.
Second appellation is related to oscillatory movements.

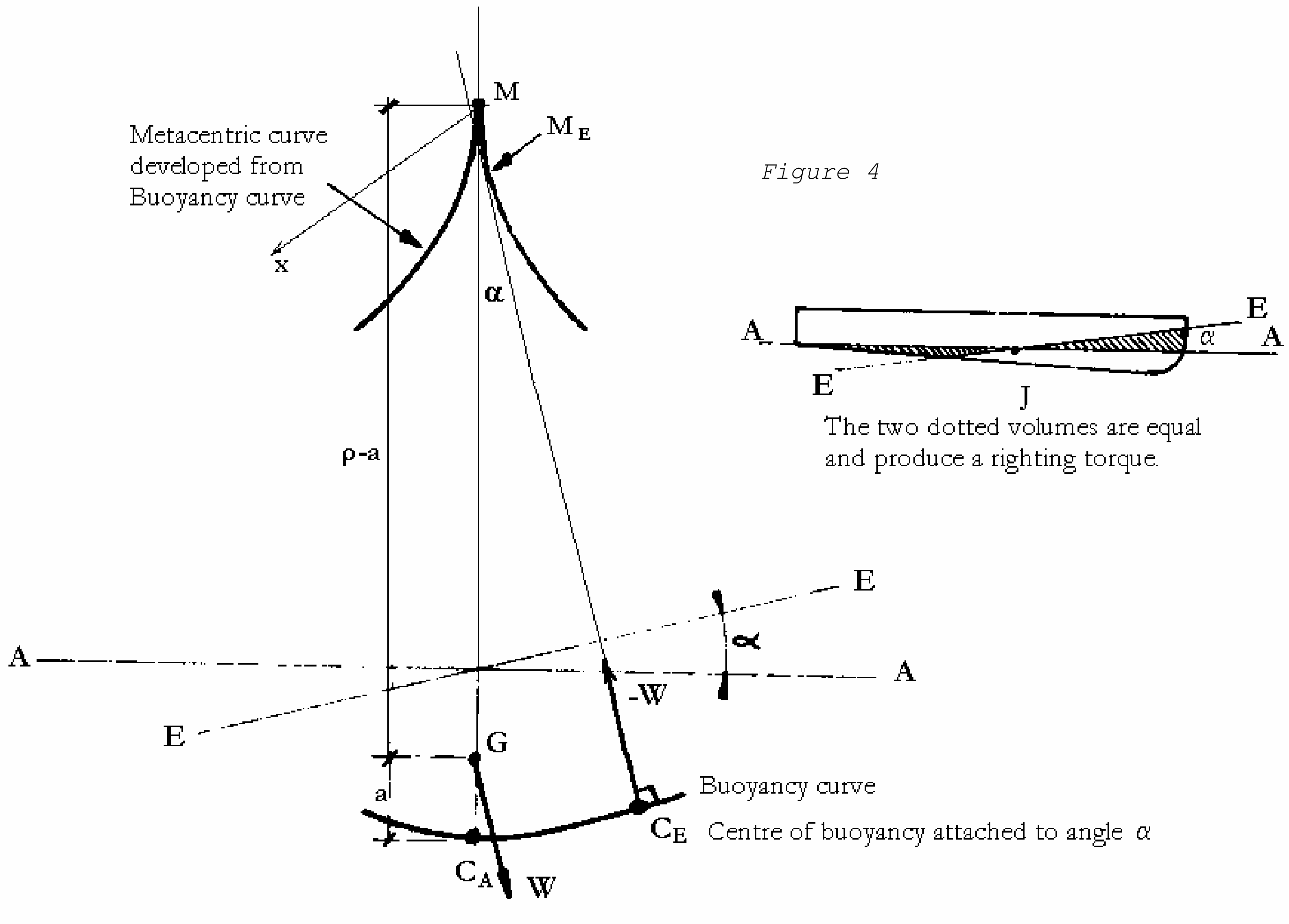
Those movements are controlled by both actions of wind and sea. Those actions are so complex that the movement of the boat cannot be calculated (by Lagrange equations for instance). Only certainty is that they are not independent from each other. Nevertheless elementary movements may be tentatively analyzed separately.

Champions try to get the best forward motion they can. To do so they transfer as much energy as possible from the wind to the boat; part of that energy is stolen by supporting water, by displacement of that water and by friction of the boat wetted surface; another part is lost in the other five movements of the boat or in their variations. There comes a maximum speed (otherwise it would go on increasing) when water takes all the energy supplied by wind. Boat's energy being equal to $\frac{1}{2}\, \mathbf{m}.\mathbf{V}^2$ (**m** : boat's mass, **V** : speed), it is most important that overall vector speed **V** keeps as close as possible to axis **Gy**.

The five other boat movements not only absorb energy by themselves but they also absorb energy by parasite water movements they induce.

I – 3, PITCHING around AXIS Jx

In figures 3 and 4, in order not to draw the boat three times, which would make the illustration less clear, we have shown normal waterline **AA** and waterlines **EE** and **FF** due to pitching. To understand the boat's behaviour, please turn the paper back and forth so that the waterline remains horizontal.



Let us draw what happens.

Figure 3 shows the normal water line **AA** and another line **EE** forming angle α with **AA**, **G** the centre of gravity, $\mathbf{C_A}$ and $\mathbf{C_E}$ the centres of buoyancy (where Archimedes thrust is applied) for each one of the two waterlines ; **W** is the boat's mass equal to the buoyancy ; **m** being the boat mass : **W=mg**. The curve along which **C** moves is the **buoyancy curve** and **CM** remains at right angles to it. Thus lines **CM** remain tangential to a curve called **developed metacentric** (developed from buoyancy curve).

The displaced volume of water **V** contained between the water plane and the wetted surface of the hull remains constant, its mass being **W=mg** .
For small inclinations, the points $\mathbf{M_E}$ remain close to a point **M** which is called **metacentre** (this metacentre is also the instantaneous rotation centre of immersed volumes), **CM** being the longitudinal metacentric radius ρ_x ; Since **M** is above **G** , the torque produced by Archimedes thrust and boat mass tends to bring the boat upright, that is to bring back line **AA** towards line **EE**. This torque is applied to axis $\mathbf{J_x}$ and for small pitching oscillations α, its value is :

$$\mathrm{mg}\times\mathrm{GM}\times\sin\alpha = \mathrm{mg}\left(\rho_{\mathrm{x}} - a\right)\sin\alpha \approx \mathrm{mg}\alpha\left(\rho_{\mathrm{x}} - a\right)$$

The equation governing $\alpha(\mathbf{t})$ is (**t** being time, with $\dot{\alpha} = \frac{d\alpha}{dt}$ and $\ddot{\alpha} = \frac{\partial^2\alpha}{\partial t^2}$) :

$$I_{Jx}\ddot{\alpha} + mg\alpha\left(\rho_x - a\right) = F(\alpha,t) \quad \text{with} \quad I_{Jx} = I_{Gx} + mb^2$$

where ***F*** is the action of wind and sea. Supposing that the wind approximately equal the resistance of sea, and thus neglecting ***F***, we find that

$$\alpha = A\sin\omega t \quad with \quad \omega = \sqrt{\frac{mg(\rho_x - a)}{I_{Jx}}} \quad (\omega \text{ being the pulsation})$$

but, as $I_{Jx} = I_{Gx} + mb^2$, writing $I_{Gx} = mr_x^2$,

where $\mathbf{r_x}$ is called radius of gyration around axis **Gx,** $\quad \omega = \sqrt{\frac{g(\rho_x - a)}{b^2 + r_x^2}}$

The period of free pitching oscillations is : $\quad T = 2\pi/\omega = 2\pi\sqrt{\frac{b^2 + r_x^2}{g(\rho_x - a)}}$

The energy associated to the movement is : $\quad E = \frac{I_{Jx}}{2}A^2\omega^2 = mg(\rho_x - a)A^2/2$

The term $b^2 + r_x^2$ has disappeared.

When the positive action of wind is opposed to an equal and negative action of sea, that is to say when function **F** may be neglected, then the **boat's moment of inertia (or her masses distribution) has no effect on its free pitching energy**. But the centre of gravity position has one through term **a.**

In a Finn, ρ_x measures about **12** m, boat's mass being equal to **145** kg
The Finns I was able to inspect had radius of gyration $\mathbf{r_x}$ comprised between **1,12** m and **1,34** m and distances **a** comprised between **8** and **17** cm.
The energy associated (boat alone) would then be comprised between

$$E = A^2 g \times \frac{145}{2} \times 11.83 = 8414 A^2 \ Newton \times m$$

and

$$E = A^2 g \times \frac{145}{2} \times 11.92 = 8478 A^2 \ Newton \times m$$

The ratio is **1.008**. The influence of **a** is certainly less than the helmsman's mass and position.

Anyhow, any pitching energy due to helmsman's movements, for instance, will be taken from forward motion energy.

In the above demonstration, all other movements else than pitching have been ignored. Yet they are more or less connected together : the boat itself creates its wake and waves ; thus sea never appears flat to the boat and the metacentre position is altered according to pitching angle α.

Nevertheless it appears that, on a smooth sea, there is no direct connection between pitching and radius of gyration or masses distribution.

I – 4, ACTION of SEA ; WAVES MOTION

This motion is still poorly understood. Therefore we will select the simplest theory formulated by GERSTNER. (*Fig.5*)

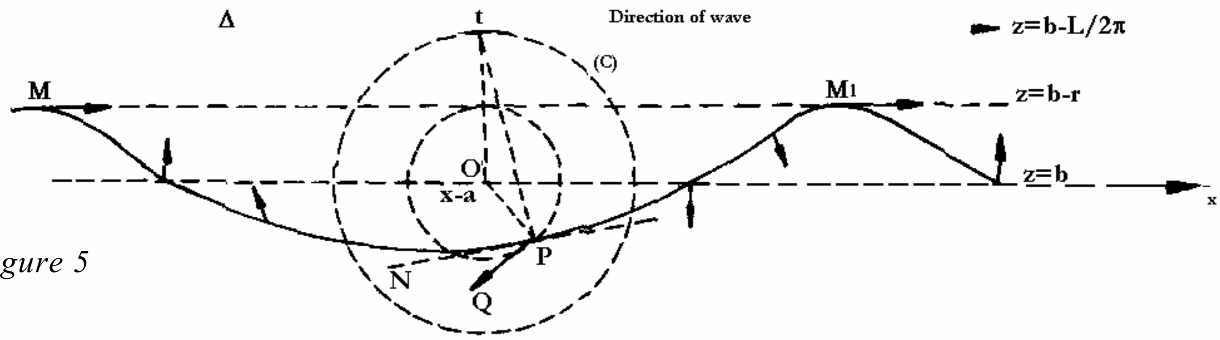


Figure 5

The liquid particles **P,** which would be set at same level **z=b** when at rest, are situated on a trochoïdal curve which is produced in following way :
a circle **C,** centre of which is **O (x=a, z=b),** rolls without slipping under the horizontal line Δ set at distance **L/2π** above **z=b.** Point **P** is attached to **C** plane at distance **OP=r** so that

$$r = \frac{L}{2\pi} e^{-2\pi b/L}$$

Tangent **PQ** to that trcochoïd is perpendicular to **tP** and **t** is the instantaneous centre of rotation.
P takes a time **T** to go around the full circle and to come back at initial position, this time being the **wave period**. Rotation speed of **P** is $\omega=2\pi/T$. If the waves move towards **Ox** , the particles rotate clockwise.
That shows the speed direction of the particles at each point.

The surface particles are interesting to follow ; we see immediately that at wave crest, those particles always move in the direction of the waves.
On the lee side of the wave crest, the movement is upwards.
On the windward side, it is downwards (Man overboard must face waves for safety, not to be overturned face down).

The period **T** and the wave length **L** are connected by $T^2=2\pi L/g$ (**g** being gravity acceleration). The speed of wave propagation **c** (celerity) $c=L/T=gT/2\pi$; it is also the speed of particles at wave crests. It is therefore the added ground speed of a boat surfing a wave.

Numerically :

Period **T** (in seconds)	Wave length **L** (in metres)	Celerity **c** (in metres per sec)
1,7	4,50	2,6
2	6	3,1
4	25	6,2
6	56	9,4
8	100	12,5
9	126	14,1

When the waves begin to break it is because the crest particles speed reach **c** value.
Therefore watch out for the distance **L** between crests, have confidence in GERSTNER and you will surely know already how advantageous it is to remain on the wave crests on a reach and on a run !

I– 5, EFFECT of WAVES upon BOAT

We are going to examine the boat in its most difficult behaviour, i.e. sailing against waves and wind.

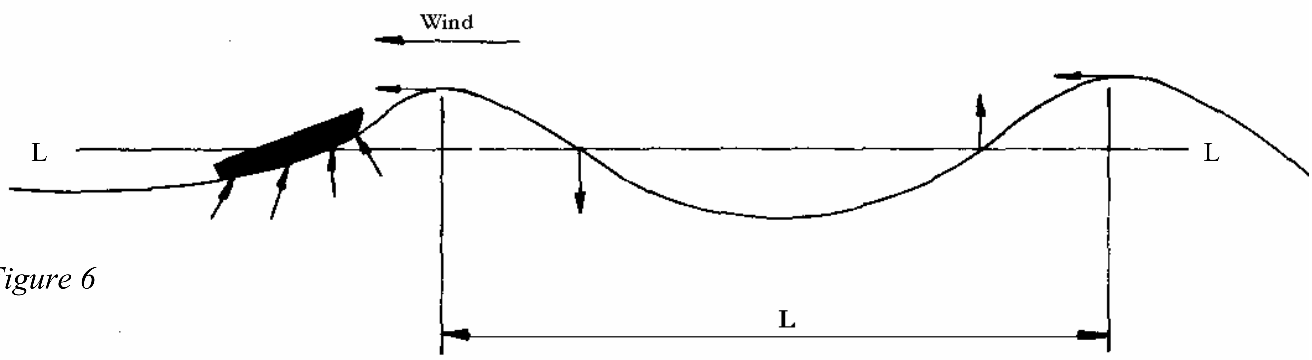
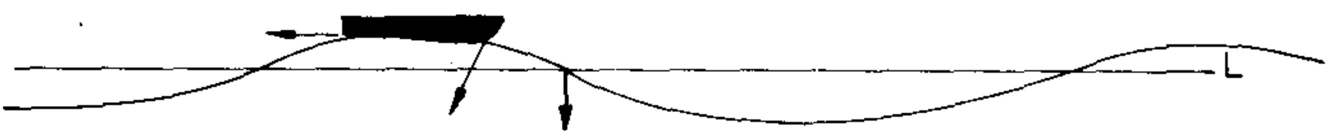
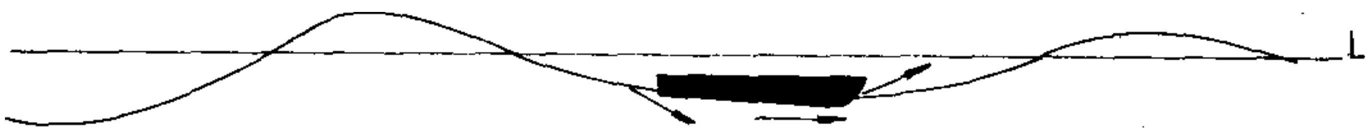


Figure 6

Fig. 6 - The boat climbs the wave, the water particles lift her and help her to move to level **LL** ; then they slow her down, more and more, up to the crest. The transom is the last part to be lifted by the particles. During all that time, the lifting motion converts the kinetic energy of the boat into potential energy and contributes to her slowing down.



Fig, 7 - The boat has climbed the crest; she has been slowed down fully and the water particles will draw her to the bottom of the swell at an increasing rate ; the stem is first drawn down. The potential energy decreases and the speed increases.

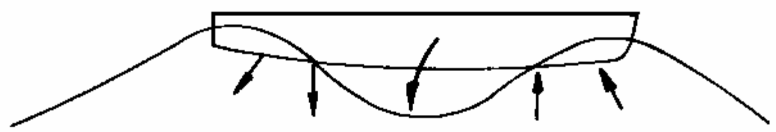


Fig, 8 - Once it has passed line **LL** the boat straightens up, its speed has reached a maximum at the bottom of the wave, it being helped by the water particles.

In conclusion, the swell transfers three elementary motions to the boat :

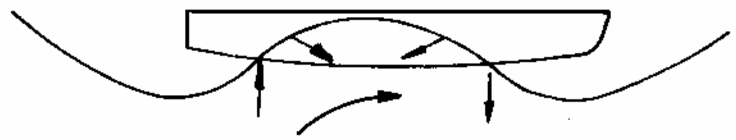
- a positive or negative forward motion which has an oscillatory look,
- an up and down motion coupled to previous one,
- a pitching motion linked to previous ones.

Those elementary movements apply resisting and oscillatory actions to the boat. According to general physical laws, they tend to couple all elementary movements between themselves and to make them more or less oscillatory.
We demonstrate below that forced pitching motion absorbs an energy linked with the design of the boat and with its masses distribution.



Positive pitching

Figure 9



Negative clockwise pitching

Fig. 9 illustrates what happens to a boat in a chop and shows that the forced pitching due to the swell has quite a serious effect. Theory tells that the pitching motion of a boat is split up into a free oscillation of period $\mathbf{T_L}$ relative to water, and a forced oscillation due to the swell, having a period $\mathbf{T_H}$, so that :

$$\alpha = \varphi_L \sin \omega_L t + \varphi_H \frac{T_H^2}{T_H^2 - T_L^2} \sin \omega_H t$$

Period $\mathbf{T_L}$ is small compared to that of the swell ($\mathbf{T_L}$ =0,6 sec whereas $\mathbf{T_H}$ =2 sec for a severe chop). Also the amplitude φ_L of the free oscillations remains small and α may be approximated thus :

$$\alpha = \varphi_H \frac{T_H^2}{T_H^2 - T_L^2} \sin \omega_H t$$

$\mathbf{T_H}$ is much greater than $\mathbf{T_L}$ so that it would seem that α remains close to

$$\alpha = \varphi_H \sin \omega_H t \quad \sin ce \quad \frac{T_H^2}{T_H^2 - T_L^2} \cong 1$$

The boat should quietly follow the swell since her free pitching movement has been neglected. That forced pitching absorbs energy.

At once two annoying events appear :

- $\mathbf{T_H}$ is a multiple of $\mathbf{T_L}$; there is resonance and the amplitude grows out of all proportion ; should the swell be regular, it is sufficient to change the heading slightly to escape resonance, but from time to time one may be surprised by variations in $\mathbf{T_H}$ making $\mathbf{T_H}$ a multiple of $\mathbf{T_L}$ and causing a sudden lurch.

- The variations of $\mathbf{T_H}$ are approximately equal to $\mathbf{T_L}$ and the helmsman needs all his skill to avoid the impacts developing into continuous slamming. In this summary about wave motion we have seen that :

$$T_H^2 = \pi L / g$$

therefore a variation $\mathbf{dT_H}$ corresponds to a variation $\mathbf{dL}$ in accordance with the relation :

$$dT_H = \pi \frac{dL}{gT_H}$$

For a wave length of **6** metres, we have seen that

$$T_H = 2 \text{ sec}$$

$\mathbf{dT_H}$ could be equal to the boat free period $\mathbf{T_L} \cong$ **0,6 sec.** , if

$$dL = \frac{gT_H}{\pi} dT_H = c\,dT_H = 1.85 \ m$$

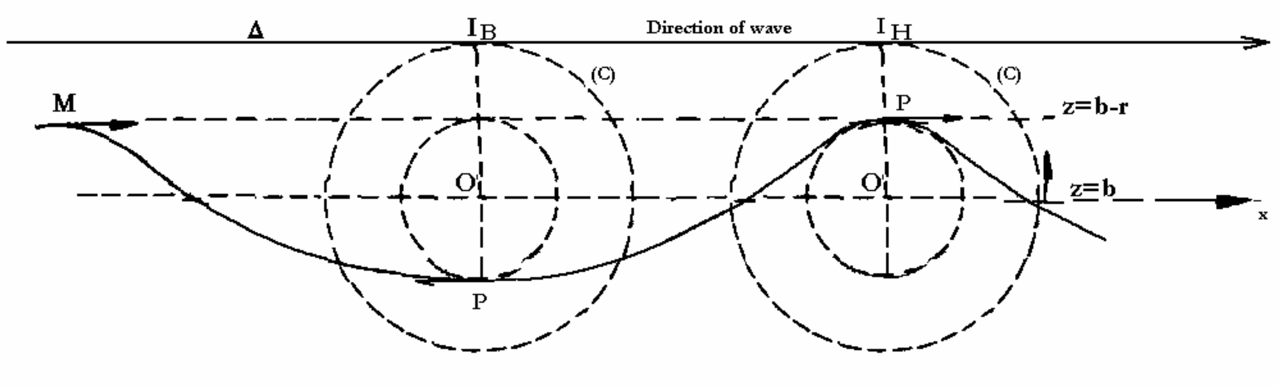
$\mathbf{dL}$ is the variation in the wave crest spacing. It is normal to see a wave length vary by **30%** since the choppier the sea, the closer the waves.

For a wave length of **25** metres we would have **dL = 6,2 x 0,6 = 3,72 m**
this corresponding to a variation of **15%** but there the helmsman has enough time to anticipate the impact and to change the boat's heading thus varying the wave length on which the boat travels.

Yet things are not that simple.

Action of water particles :

Let us consider a boat whose quick works are close to surface.



At the bottom of a wave, the boat approximately moves around point $\mathbf{I_B}$ with a radius $\mathbf{r_B} \cong \mathbf{I_BP}$ such that

$$r_B = b + \frac{L}{2\pi} e^{-2\pi b/L}$$

On the crest of a wave the boat moves around a point $\mathbf{I_H}$ with a radius $\mathbf{r_H} \cong \mathbf{I_H P}$ such that

$$r_H = b - \frac{L}{2\pi} e^{-2\pi b/L}$$

There appears an energy induced by friction of water particles against the hull :

$$1/2\, m\omega^2 \left(r_x^2 + IG^2\right)$$

IG varying between $\mathbf{r_B}$ and $\mathbf{r_H}$.

In a chop where **IG** is small, those losses of energy linked with masses distribution by parameter $\mathbf{r_X}$ are relatively greater than in a large swell.

General action of swell

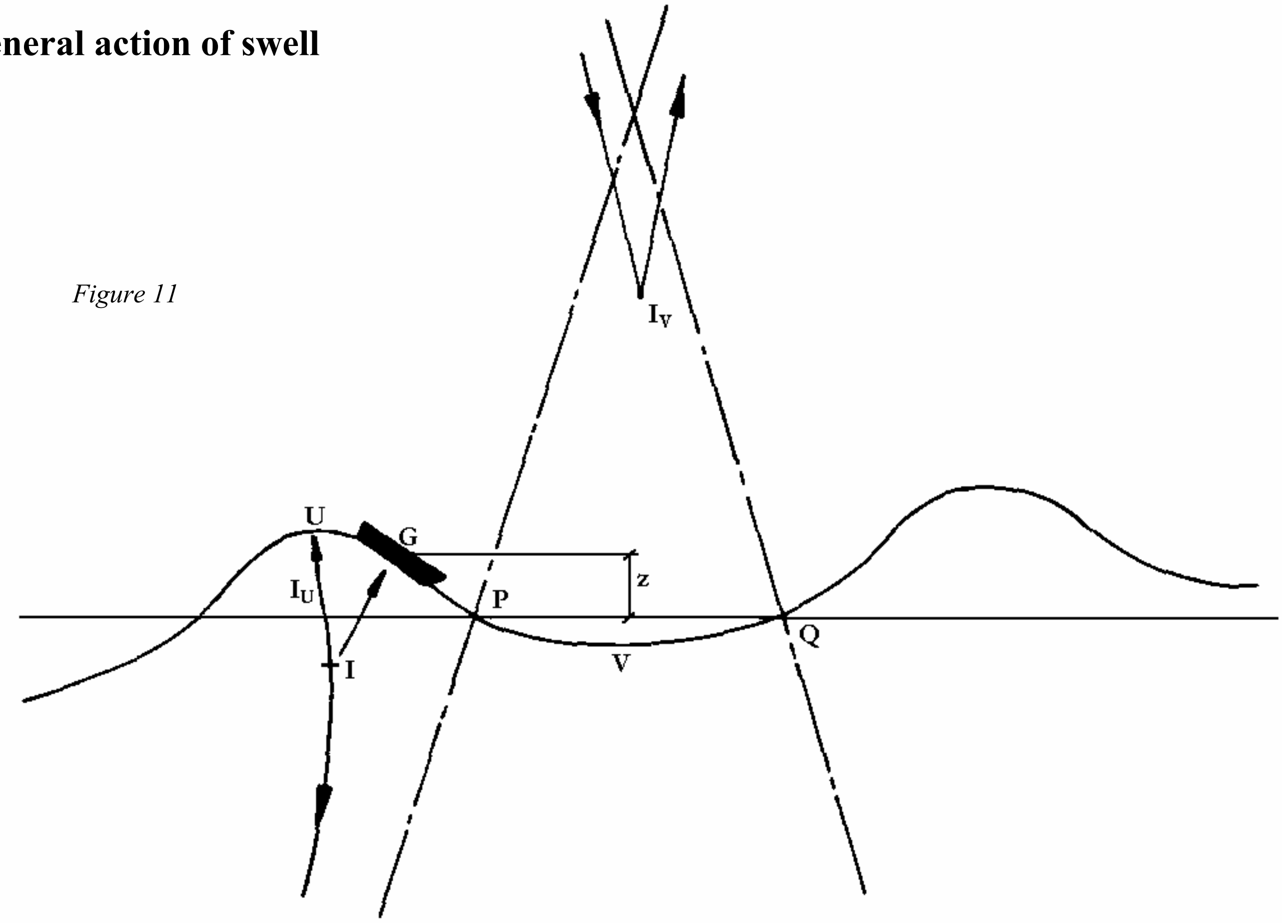


Figure 11

On the crest of a wave, the boat pitches around a point $\mathbf{I_U}$, then the centre of rotation moves away towards infinity as the boat passes inflection point **P**, the slope passing through an extreme an the curvature through naught .When the boat moves along **PVQ,** the centre of her forced pitching returns from infinity to point $\mathbf{I_V}$ and then goes back again to infinity (Fig 11).
The pitching energy supplied by the swell to the boat is :

$$1/2\, m\omega^2 \left(r_X^2 + IG^2\right)$$

Point $\mathbf{I_V}$, because of the shape of the swell, is much farther from the boat than point $\mathbf{I_U}$.
Actually, $\mathbf{I_U}$ may undergo large variations and come to a minimum when waves come to break. If the boat was reduced to a point, $\mathbf{I_U}$ would be on the crest. In fact, it is lower as the boat is supported over some length (Fig 12).

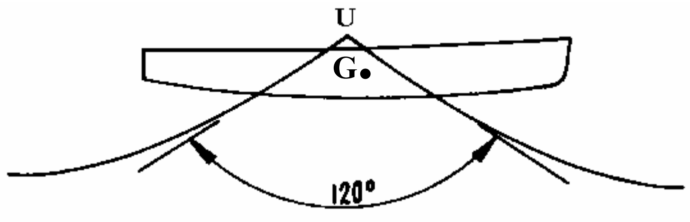


Figure 12

It also appears that the lower **G,** the less **I_UG.**

So at wave crests, forced pitching is more important and it is a fast one. It does not only induce a lost of energy but also an **ugly behaviour of sails** the top of them moving faster than the foot.

I – 6, YAW around axis G_z

The above action of waves originates another forced oscillation around an axis parallel to **G_z**, located between bow and front edge of centerboard.

When beating, the helmsman heads up to climb the wave (as his speed is less and apparent wind too), then he bears off down the wave (as apparent wind is stronger and veering). That yaw movement is continuous and oscillatory. It is controlled by the rudder. The greater the yaw energy, the greater the resistance of the rudder and the greater its slowing down action.

Another recommended yaw action is the permanent search for the lowest water near the bow. Champions know that well as soon as sea becomes choppy.

I – 7, EFFECT of MASS DISTRIBUTION and of CENTRE of GRAVITY HEIGHT

Masses distribution is characterized by the radius of gyration $\mathbf{r_X}$ of the boat.

We have already seen that it had no effect on the boat's free oscillation period $\mathbf{T_L}$. But in the case of irregular forced oscillations it has a great importance. If ω is the maximum angular speed of a forced oscillation, the energy absorbed by the boat in pitching is :

$$1/2\, m\omega^2 \left(r_x^2 + IG^2\right)$$

Taken from forward energy, it is quickly spilled by water damping or by wave impacts.

Let us tell it another way : **when a boat crosses a wave, she is induced to pitch with a forced amplitude A ; A is higher when $\mathbf{r_X}$ is higher and also when centre of gravity is higher. Pitching energy is taken from progressing energy because there would be no pitching should the boat move backwards as fast as the waves. After having passed the crest, the initiated pitching of amplitude A becomes a free pitching soon damped by water frictions.**

As is known by physics laws, resistant actions use to couple elementary oscillatory movements, which is another way of spilling forward energy through its partial oscillatory behaviour (boat progresses faster down waves and slower up waves).

In order to reduce energy losses as much as possible, $1/2\, m\omega^2 \left(r_x^2 + IG^2\right)$ should remain as low as possible.

The angular speed ω is derived from swell ; **m** is fixed by boat design; but $\mathbf{r_x}$ and **IG** may vary in some degree if not controlled. The radius of gyration should be kept as low as possible and therefore all possible masses of materials should be gathered near the centre of gravity.

When beating through wave crests, **IG** should also be as low as possible which leads to lower centre of gravity.

The loss of energy due to yaw is directly linked with the $\mathbf{r_Z}$ radius of gyration which is also characteristic of masses distribution.

In the Finn class I was able to observe the two following extremes : $\mathbf{r_x}$=1.12 m and $\mathbf{r_x}$ =1.34 m.

Neglecting **IG**, the energy stored at same angular speed ω varies by

$$\frac{1.34^2 - 1.12^2}{1.12^2} = 43\%$$

Indeed, the **difference is a great one** !

I – 8, CONCLUSION

The boat should move as lightly as possible in a swell. To do this, one tries to gather as much as possible of the matter near the centre of gravity.
When we are carrying boats on shore, (and we all have done that), we sometimes feel that for a same given mass some boats are lighter and more easily handled than others. It is simply because the seemingly heavy ones have too much matter in the ends and that it is easier to handle concentrated matter then distributed matter.
The waves supporting your craft have the same feeling and will consequently bear the boat lightly or heavily. This heaviness is characterized by the expression :

$$1/2\, m\omega^2 r_x^2$$

where the radius of gyration $\mathbf{r_x}$ representing the mass distribution is expressed as a square whereas the total mass **m** only comes in linearly.

Similar problem with the yaw action of the rudder.

For both movements **m** also appears in the propulsive power expression $1/2\, mV^2$

In order that this energy be retained in spite of waves, it is better to keep an appreciable value for **m**. As regard Finns, that value is quite high and allows them to go through quite a steep chop. Those two main reasons will encourage to reduce $\mathbf{r_x}$ or $\mathbf{r_Z}$ rather then **m**.

Last, it appears that masts, centre of gravity of which is far from overall centre, have a great influence. Multihulls which have high masts know well how important it is to lower mast mass and all racing multihulls are now equipped with carbon masts; carbon has a lower density but also higher yielding stresses, which allows thinner and lighter masts.

II PART TWO
CONTROL of the MASS DISTRIBUTION

II – 1, NEED for CONTROL of MASS DISTRIBUTION

That control had been asked by IYRU, as mentioned in introduction and I believe that I have demonstrated that the desire of helmsmen to lighten the ends of their boats is not a passing mood.

The mass distribution of a boat a seems to be even more important than the mass itself. To control this distribution there is no available instrument and for a long time people have tried to find a mean of control in as simple a manner as one can control mass i.e. with a scale.

When mass distribution could not be measured, it could be observed that same helmsman and boat would win all regattas over the world. Since it may be measured, nobody may tell who will win.

Still now, 30 years after, no Finn helmsman would sail a boat over minimal radius of gyration and minimal height of centre of gravity. Those are now measured at factory.

I do not know how the matter is ruled in other classes. But many ones, such as Flying Dutchmen, 505, 470, Europe Moths, Dragons …. had asked me for apparatus and means of calculations.

II - 2, FORMER CONTROLS CARRIED out in the FINN CLASS

Finn rule n° 3 told that the mass distribution should be as close as possible to that of a wooden boat with a hull of constant thickness throughout.

The shifting of the centre of gravity by moving material was further restricted by a certain number of other rules such as :

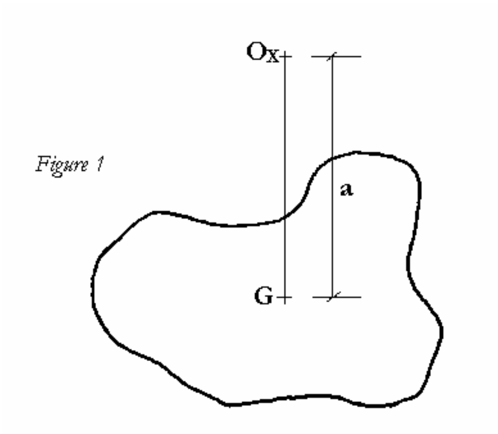
> *17) - Laminated wood and plastic construction together is not allowed.*
> *19) - The hull material must not contain trapped air cells.. Hollow reinforcing pieces must be left open et their ends. The hull thickness should always be greater then 3 mm. Mulls may have to be drilled in order that checks can he made.*

20) - Wooden hulls should be at least 9 mm thick. Moulded wood should be made Lip of constant density layers.
45 - 47 - 49) - Extremely accurate definition of the floor boards : the material density is fixed.
65) - Height of mast centre of gravity.
79) - Restriction of openings in transom.
98) -Forbidding the carrying of items which might serve as ballast. The inclusion of light or heavy materials in the construction is not allowed..

We had always known that those rules left some latitude to builders. Furthermore they were often imprecise and advantage was sometimes taken from that lack of precision. to hide some trickery (alas it had been known to happen !)

Many Class owners' associations had tried for a long time to find serious ways of measuring the distribution of mass. In fact several methods had been known for a long time but putting them into practice would have involved calculations that would frighten the measurers and be quite unusable in regatta conditions.
Now computers and even pocket ones have allowed us to develop following procedure which has been found to be particularly simple.

II – 3, MEASUREMENT SYSTEM introduced by FINN CLASS



Let us consider an object **S** oscillating around axis $\mathbf{O_X}$

- with a radius of gyration $\mathbf{r_X}$ (I have shown in part one that this radius of gyration was a characteristic of mass distribution),
- with a centre of gravity **G,**
- at a distance **a** from **G** (*Fig 1*).

We are dealing with a composite pendulum, the oscillation period of which around axis $\mathbf{O_X}$ being (according to Huighens) :

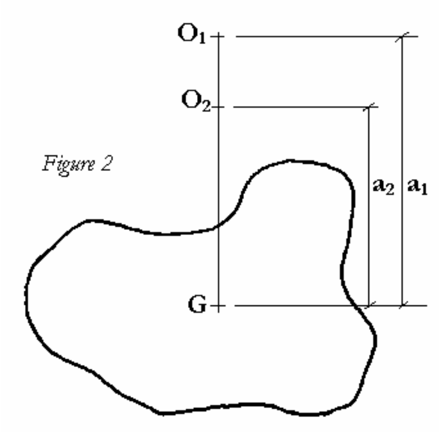
$$T = 2\pi\sqrt{\frac{\left(a^2 + r_X^2\right)}{ag}} \quad (\mathbf{g} = \text{gravity acceleration})$$

(We should note that those oscillations have nothing to do with those of a boat on water).

If the position of the centre of gravity is known, **a** is known and by measuring period **T**, we immediately find $\mathbf{r_x}$. Actually **a** is difficult to assess and we have two unknowns : $\mathbf{r_x}$ and **a**. If the object under examination is being made to oscillate successively around two parallel axis $\mathbf{O_{1X}}$ and $\mathbf{O_{2X}}$ separated by a known distance **b**, we can measure two oscillation period $\mathbf{T_1}$ and $\mathbf{T_2}$ around those two axis and we have two equations which allow to calculate $\mathbf{r_x}$ and **a** (*Fig 2*).

$$T_1 = 2\pi\sqrt{\frac{a_1^2 + r_x^2}{a_1 g}} \qquad T_2 = 2\pi\sqrt{\frac{a_2^2 + r_x^2}{a_2 g}}$$

with $a_1 - a_2 = b$



II – 4, CALCULATION of a_1, a_2 and r_X.

In 1970 years we would draw graphs delivering solutions of above equations. Accurate drawing of those graphs had been made possible by use of electronic drawing machines driven by computer. Thus we could obtain values of $\mathbf{r_x}$ and **a** with an accuracy of 0.1 mm. (*Fig 7*).

Nowadays we may use programming pocket calculators which deliver the searched results.

We find that $a_1\left[\frac{g}{4\pi^2}\left(T_2^2 - T_1^2\right) + 2b\right] = b^2 + T_2^2\frac{bg}{4\pi^2}$ hence $\mathbf{a_1}$

and that $r_X^2 = \frac{a_1 g T_1^2}{4\pi^2} - a_1^2$ hence $\mathbf{r_x}$

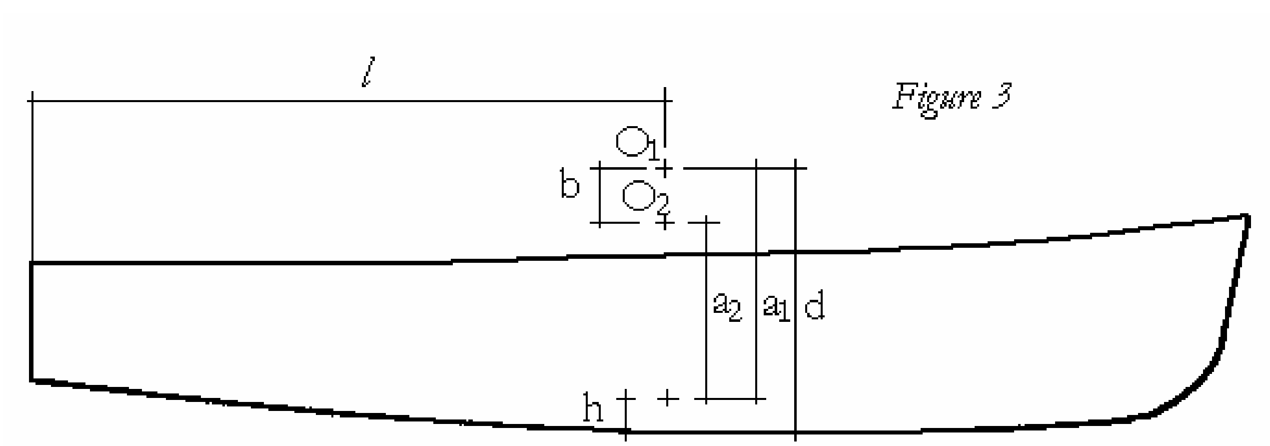
Those calculations may be automated with a pocket calculator.

For each class of boats, that they be centre board boats or keel boats, it appears that different values of **b** must be used so as to get maximum precision.

II – 5, POSITION of the CENTRE of GRAVITY

Moreover the method provides the position of the centre of gravity **G** .
As **G** is in line with $\mathbf{O_1}$ and $\mathbf{O_2}$, its fore and aft position may be located by the measurement of **l** (*Fig 3*). Then we will measure distance **d** between $\mathbf{O_1}$ and the underneath of hull (For Finns, it happens that $\mathbf{O_1}$ is always situated above the centreplate box and with a rule down the inside of that box one can easily measure **d** . The vertical position of the centre of gravity will thus be known from dimension **h** such that

$$h = d - a_1$$



II – 6, SETTING up AXIS 0_1 AND 0_2 in PRACTICE

This is what took me the longest time. In the end, I found that the simplest way was to support the boat by the rubbing strakes. Therefore I made two brackets as illustrated in Fig 4 on which the boat could be hung.

These items are cut out of a single 6 mm thick steel sheet and may to made by any metal worker. They weigh a little more then 1 kg each and combine with the boat's mass in its pitching motion, but being very close to the centre of gravity **G** they hardly affect the radius of gyration $\mathbf{r_x}$ (*Fig.4*).

Steel parts in contact must be hardened by cementation.

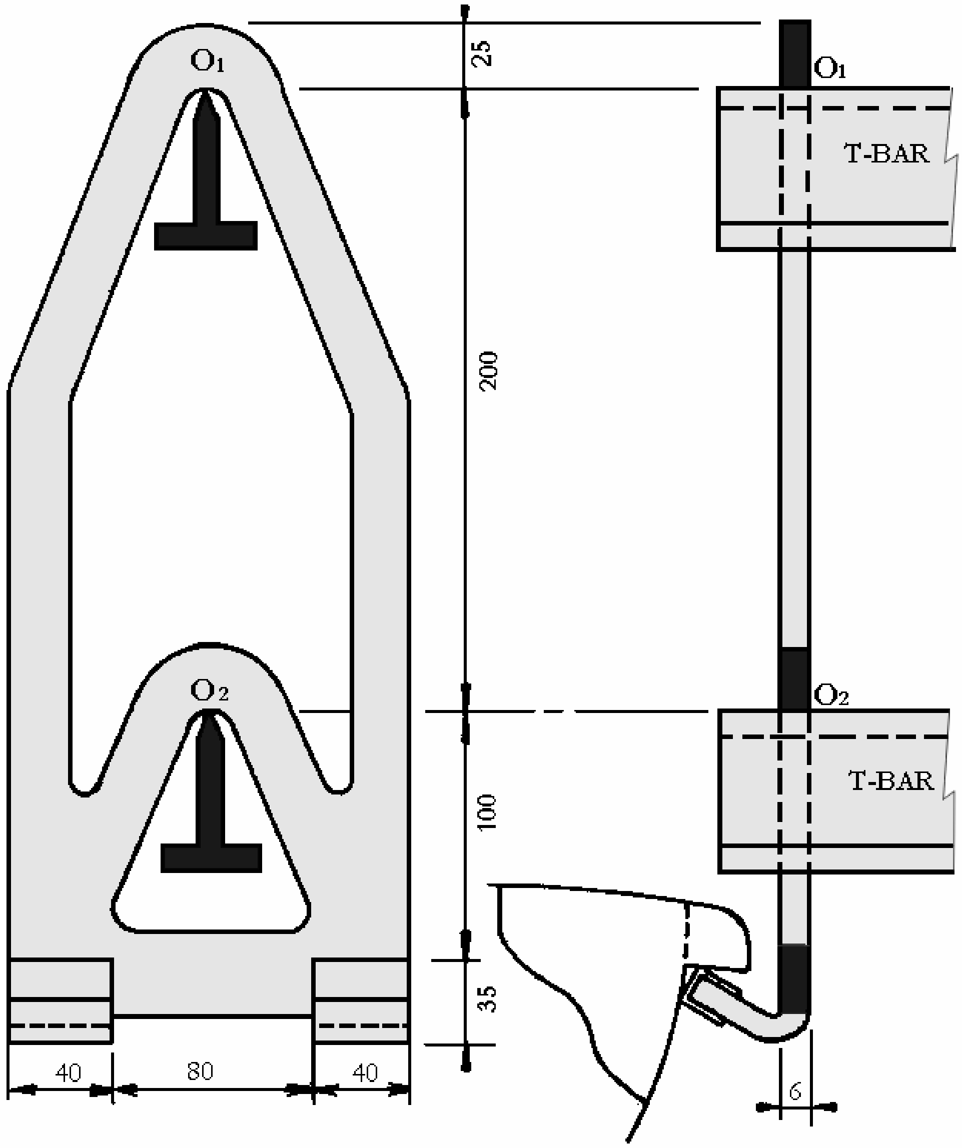
Figure 5 shows how to set up the apparatus.
Two trestles bear steel pivots; those pivots are made of T bars sharpened into knife edges which are meant to be the oscillating axis.
Two countermasses are attached on either sides of the trestles (unless the latter are fixed to the floor; which allows to retain them in position at all times with the pivots well lined up).

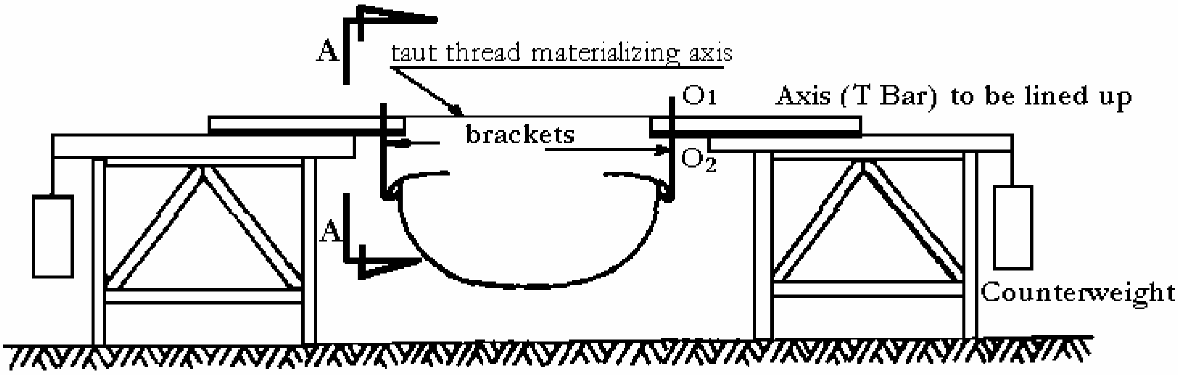
All you have to do is to bring the boat between the trestles and hang it onto the pivots, either at $\mathbf{O_1}$ or at $\mathbf{O_2}$ and to shift it slightly until it is approximately horizontal when at rest.
In a sheltered place the oscillations are damped in approximately 100 periods, making a perfect pendulum.

I have tried to offset the pivots by combining the rotation around axis $\mathbf{O_1}$ or $\mathbf{O_2}$ with a transverse rocking motion. We took measurements at CASCAIS Gold Cup (1970) in the open and in a strong wind (this is actually not recommended). I have always found that the lengthwise pitching period was not affected and remained constant within a few hundredths of a second.

Nevertheless, according to pendulum theory, the oscillations must keep small amplitudes and must be damped as less as possible.

Figure 4





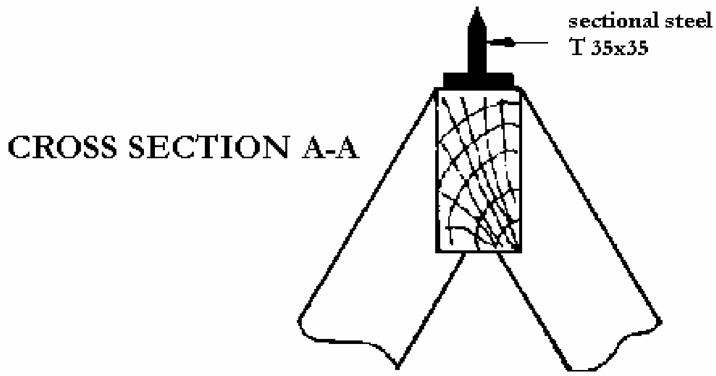


Figure 5

II – 7, MEASUREMENT OF PERIODS T

This is the operation requiring the most care.

Present Finn rules tell :

MASS DISTRIBUTION AND CENTRE OF GRAVITY PRACTICE

It is essential that the measurements be made in a sheltered place. The boat shall be hung from the brackets on axis O_1, O_2, and the periods of oscillation T_1, T_2 measured.

Plot the position with co-ordinates T_1, T_2 on the graph, and read off *the values for* ***a*** *and* $\boldsymbol{r_X}$ *from the curves. The distance* ***l*** *is measured parallel to base line from Station 0 to axis O_1....*

The distance ***d*** *can usually be measured from axis 0_1 to the underneath of the hull (excluding keel band) by means of a rule or tape passed down through the centreboard box. If this is impossible, use the principle shown in diagram .... It is wise to provide a protection under the boat but the boat shall not touch anything while oscillating. The oscillations shall be small, but should not become damped in less than about 100 periods. There shall be no twisting oscillations about a vertical axis There shall be no movement of the supports. The measurement of periods T_1 and T_2 requires most care. It is recommended to operate in the following way: two time keepers stand on either side of the boat, they shall start their stopwatches when the boat passes the rest position which is made easier with two rods placed opposite each other as in fig 6 : they count ten pitching periods and if they get the same result within 0.1s, the measurement is satisfactory (the result being thus 0.01s accurate).*

Stopwatches accurate to 0.05s, shall be used. If a stopwatch only accurate to o.1s is used, twenty pitching periods shall be measured.

On cover, we give a copy of the graph included with each owner's certificate book

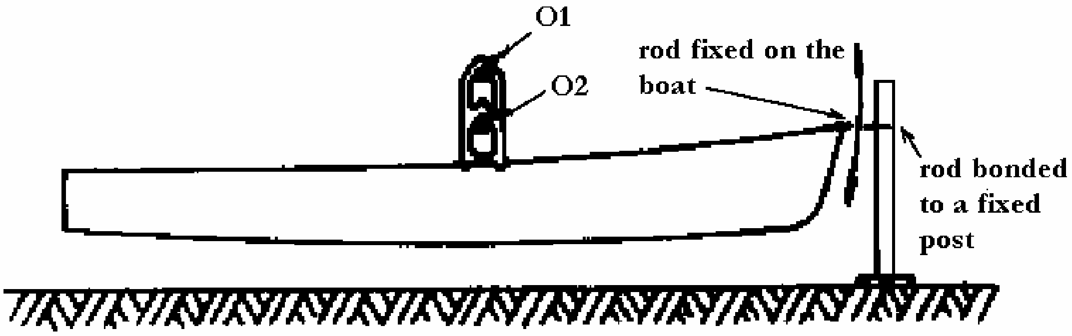


Figure 6

At Gold Cups (Finn world championship) over 100 boats may be measured in less than one day.

II – 8, ACCURACY of RESULTS

We have designed an accurate and practical way of detecting the mass distribution in a boat.
The abstract of graph which can be seen on *Fig 7* shows that the value of **a** plays a prominent part in the accuracy of results.
The smaller the value of **a**, the more accurate the results. This means that the boat must be hung up as close as possible to the centre of gravity. There we have to agree to a compromise between the simplicity of the bearing apparatus and the value of $\mathbf{a_1}$.

The apparatus used by Finns, enables to bring the values of $\mathbf{a_1}$ down to about 0,45 m.
For such a value, Figure 7 illustrates the consequences of $\pm 0.01\text{sec}$ and $\pm 0.02\text{sec}$ errors in the measurement of $\mathbf{T_1}$ and $\mathbf{T_2}$. An accuracy of $\pm 0.01\text{sec}$ upon $\mathbf{T_1}$ and $\mathbf{T_2}$ leads to an accuracy of $\pm 1\,cm$ upon $\mathbf{r_X}$ and $\pm 0.5\,cm$ upon $\mathbf{a_1}$.

The comparison between an automatic measurement (which I first designed) and a hand measurement enabled me to check that a human time keeper could time with an accuracy of $\pm 0.05\text{ sec}$ (I was not expecting such an accuracy). Over ten periods he will, therefore, make a total error of less than $\pm 0.1\text{sec}$, i..e. $\pm 0.01\text{sec}$ for one period) ; $\mathbf{r_X}$ and **a** can then be gauged with the precision indicated above.

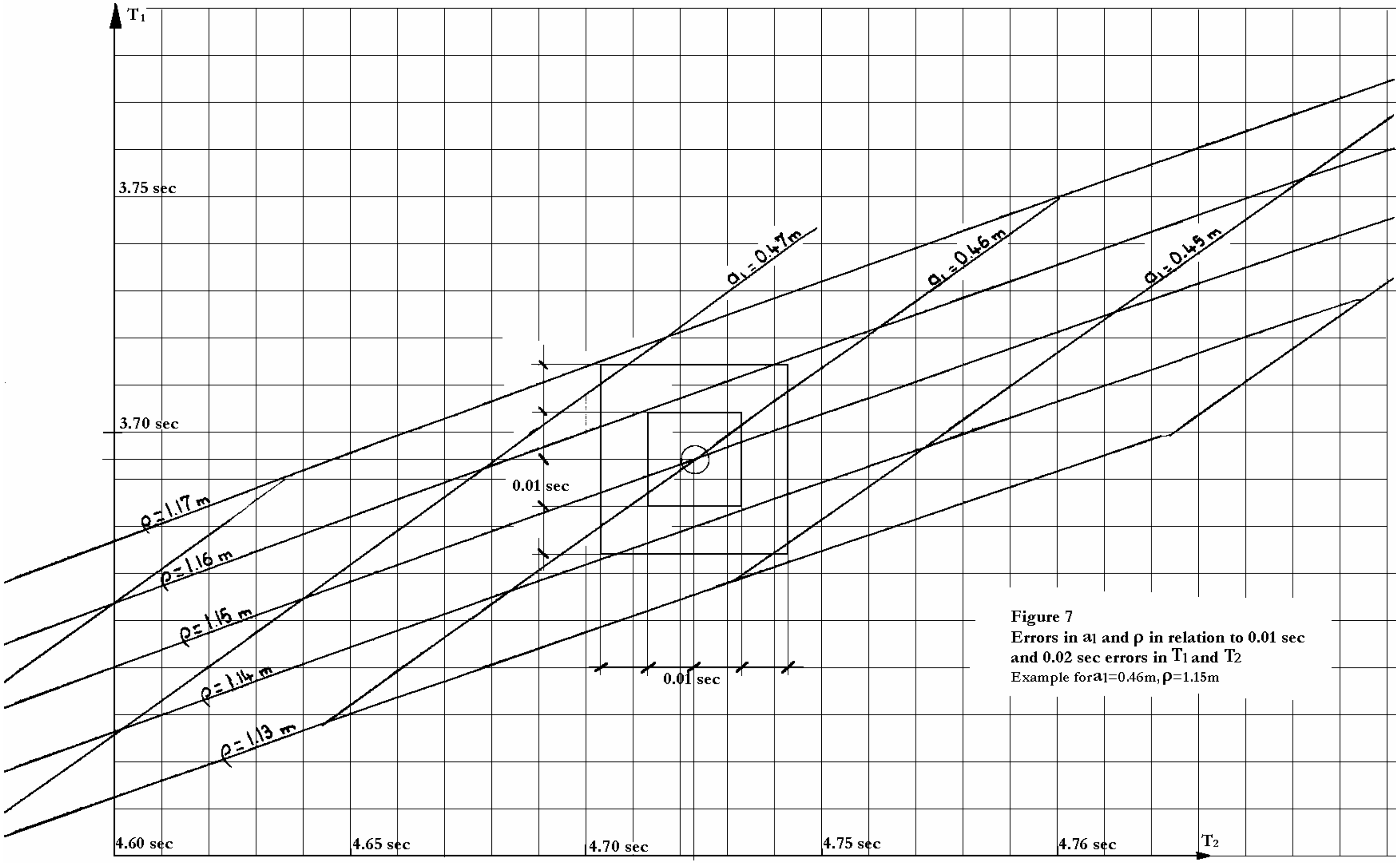


Figure 7

Errors in a_1 and ρ in relation to 0.01 sec and 0.02 sec errors in T_1 and T_2

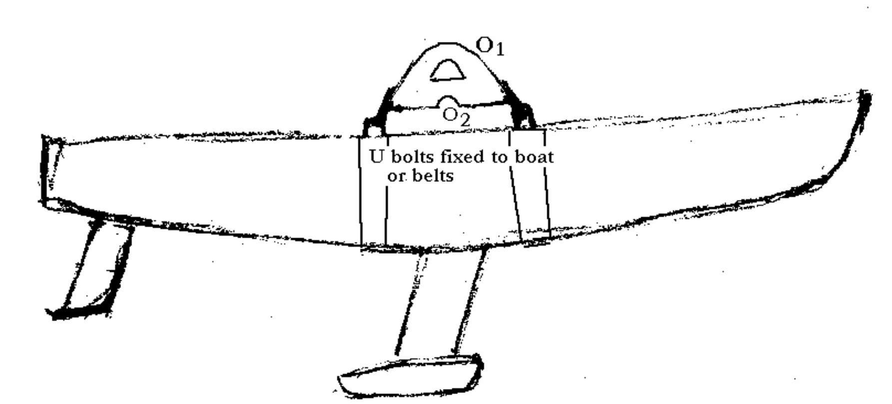
Example for a_1=0.46m, ρ=1.15m

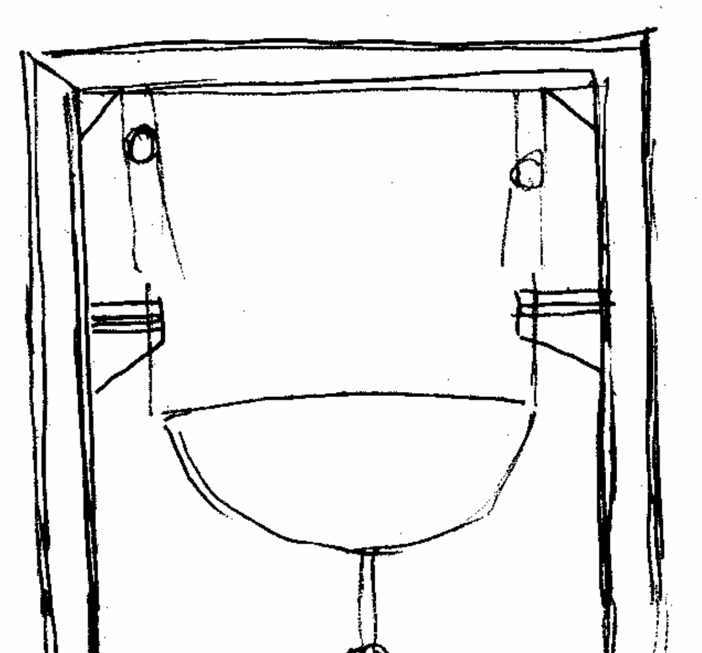
II – 9, EXTENSION of the METHOD to other CLASSES

The method is directly applicable to all dinghy classes as we already have stated. As far as I know, it as been enforced or currently used by

- Europe
- Flying Dutchman
- Yingling
- Star
- 505
- 470
- Tornado
- ……..

As regards heavy boats, it may be adapted as shown on underneath sketch although a better apparatus may be found such as sort of cradle.



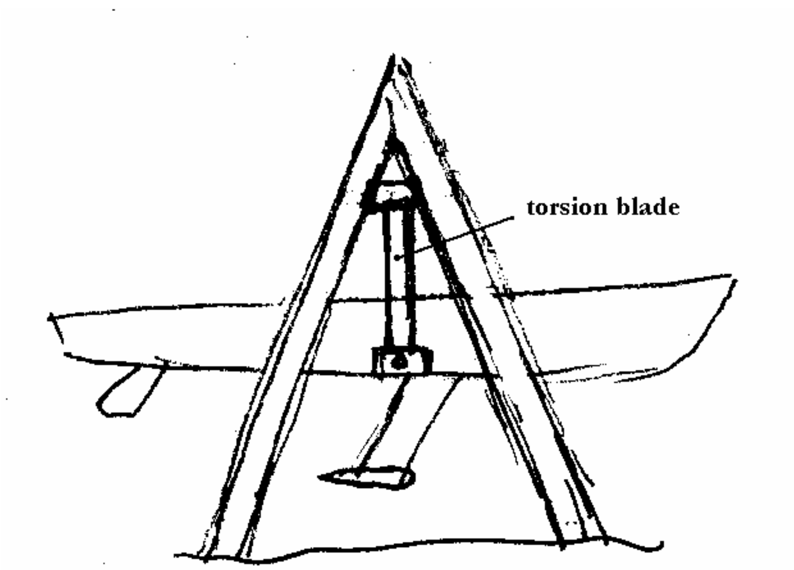


Handicap rules should take radius of gyration into account. And it would not be that complicated for builders to have those radius being measured.

One class of keel boats (Stars ?) had tried to measure yaw oscillation periods around a vertical axis, the boat being called back by a couple of springs tied at bow. But the results were not consistent depending of springs tensions and of fixations.
Furthermore the method allowed to measure the inertia $\mathbf{I_Z}$ of the boat around her vertical axis $\mathbf{G_Z}$ and it could not give the height of the centre of gravity. In a keel boat, the keel has an important effect upon the pitching energy ; the keel being close to $\mathbf{G_Z}$, its effect is little in yaw movements.
Yet the method showed to be so simple that it could turn performing by using a blade to hang the boat as on underneath sketch.
Should the torsion rigidity of the blade be **K**, the oscillatory movement of the boat around $\mathbf{G_Z}$ would be such that :

$$\boldsymbol{I_Z}\frac{\partial^2\theta}{\partial t} + \boldsymbol{K\theta} = \mathbf{0} \quad \text{with a period} \quad T = 2\pi\sqrt{\frac{I_z}{K}}$$

and the moment of inertia around $\mathbf{G_Z}$ should immediately be known ; that moment being also characteristic of masses distribution around an axis passing through **G**.
Actually two blades at right angle should be used so as to avoid parasite bending oscillations.
Blades are precise tools and are used, for instance to measure the thrust of plane engines.



At 1984 Olympics, Peter Hinrichsen used a clever way to check the yaw moment of inertia $\mathbf{I_Z}$ of Flying Dutchmen around vertical axis $\mathbf{G_Z}$. That method may easily be applied to keel boats.
Calling back moment was produced by two parallel wires suspending the boat. Should the length of wires be **l** and their distance **2d,** the yaw rigidity is found to be :

$K = 2mg\frac{d^2}{l}$, thus pulsation is $\omega = \sqrt{\frac{K}{I_z}}$ and yaw radius of gyration is $r_{z=} T/2\pi\, d\sqrt{g/l}$

How simple !

II – 10, DIFFERENT WAYS OF FINDING « a » AND « ρ » FROM MEASURED OSCILLATING PERIODS T1 AND T2

Recalling Principles for controlling Mass Distribution and position of Centre of Gravity:

The degree of concentration of the masses (or of the matter) in a boat is described by her radius of gyration. A boat with "light ends" has a short radius of gyration and moves more easily through waves.

*In Diagram 20 of measurement rules, if "**a**" is the distance from the oscillation axis 0_1 to the centre of gravity **G**, if "ρ" is the radius of gyration, and if "**g**" is the acceleration due to gravity, then the oscillating period T_1 is given with a good precision for small and little damped oscillations* by :*

$$T_1 = 2\pi\sqrt{\frac{a^2+\rho^2}{ag}}$$

*We can measure T_1 but we have two unknowns "**a**" and "ρ"; so we need two equations. Another is obtained by choosing a new oscillation axis 0_2 exactly **b=200 mm** lower. New period will be*

$$T_2 = 2\pi\sqrt{\frac{(a-b)^2+\rho^2}{(a-b)g}}$$

*Hence by measuring T_1 and T_2 we may find "**a**" and "ρ" from either underneath program or graph (to be redrawn with nowadays current tools).*

Finding "a" and "ρ" with a pocket calculator program :

It is now easier to use a pocket calculator with a program of following type:

Input T_1 **(sec)**

Input T_2 **(sec)**

Input **b=0.2 (m)**

Input **g (m/sec²)**

Calculate $k = \frac{g}{4\pi^2 b}$

Calculate $a = b\frac{kT_2^2+1}{k\left(T_2^2-T_1^2\right)+2}$

Calculate $\rho = \sqrt{abkT_1^2 - a^2}$

Show or print **a** *and* ρ **(m)**

Check program with $g = 9.80\ m/\sec^2 \quad T_1 = 3.31\sec \quad T_2 = 3.81\sec$

Result should be $a = 0.593\ m \quad \rho = 1.123\ m$

Finding "a" and " ρ " from Excel (Please click to get actual spead sheet)

	INITIAL DATA				
	$g = 9.8\ m/\sec^2$	b=	0,2	⟶ k=	1,2411845
Measurements		Results			
T1 (sec) =	3,31	a =	0,59256251	m	
T2 (sec) =	3,81	$\rho =$	1,12270514	m	

In that Excel Table, all calculations have been prepared, so that from any PC having Excel program, a *and* ρ *will immediately appear if the measured values replace the values* $T_1 = 3.31\ \mathrm{sec}$ *and* $T_1 = 3.81\ \mathrm{sec}$ *in above example.*

Finding "a" and " ρ " from graph Please Click to get graph

$T_1 = 2\pi\sqrt{\dfrac{a^2+\rho^2}{ag}}$ *may also be written* $a^2+\rho^2 = T_1^2\dfrac{ag}{4\pi^2}$ *and also*

$$\left(a - T_1^2\frac{g}{4\pi^2}\right)^2 + \rho^2 = \left(T_1^2\frac{g}{8\pi^2}\right)^2$$

Last equation is that of a circle of coordinates $a\ ,\rho$ *, radius of which being* $T_1^2\dfrac{g}{8\pi^2}$ *, centre of which being at*

$a_{c1} = T_1^2\dfrac{g}{4\pi^2}\ ,\quad \rho_{c1} = 0$.

$T_2 = 2\pi\sqrt{\dfrac{(a-b)^2+\rho^2}{(a-b)g}}$ *is also the equation of a circle of coordinates* $a\ ,\rho$*, radius of which being* $T_2^2\dfrac{g}{8\pi^2}+b$ *, centre of which being at* $a_{c2} = T_2^2\dfrac{g}{4\pi^2}+b\ ,\quad \rho_{c2} = 0$

Searched $a\ ,\rho$ *are at the intersections of the above circles which vary according to parameters* $T_1\ ,T_2$*. Those families of circles might even be drawn by hand. Interesting parts of them have been taken out to achieve attached graph.*